Matrices and Gauss Jordan Solving

$$-x + y - z = -14$$
$$2x - y + z = 21$$
$$3x - 2y + z = 19$$

Matrices!

Matrix-a rectangular array of real numbers

Row (horizontal)

Column (vertical)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{2n} \\ a_{31} & a_{32} & a_{33} & a_{3n} \\ a_{m1} & a_{m2} & a_{m3} & a_{mn} \end{bmatrix}$$

Matrices are in form m x n (m by n), where m is the number of rows and n is the number of columns.

Entries in the ith row and jth column are denoted by a_{ij} (so a_{32} is in the 3rd row and 2nd column)

* order

row x columns

Example 1

Determine the order of each matrix:

a) $\begin{bmatrix} 3 & 5 \\ 14 & -9 \end{bmatrix}$

2 x 2

c) $\begin{bmatrix} 18 & 7 & 1 \\ -64 & 2 & 0 \end{bmatrix}$

2 x 3

b) $\begin{bmatrix} 16 & 7 & 1 & 17 \end{bmatrix}$

1 x 4

Solve:

$$x - 2y + 3z = 9$$
$$-x + 3y = -4$$
$$2x - 5y + 5z = 17$$

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$$

Reduced Row Echelon Form

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x: 1 \quad y: -1 \quad z = 2$

$$\begin{bmatrix} & & & \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ ∞ Sol

$$\begin{bmatrix} & & & \\ 0 & 0 & 0 & 3 \end{bmatrix}$$ No Sol.

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -2 & 2 \\ 2 & 4 \end{bmatrix}$$

$$A - B =$$

$$5A =$$

59)

$$\begin{bmatrix} -x_1 + x_2 = 4 \\ -2x_1 + x_2 = 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -1 & 1 & 4 \\ -2 & 1 & 0 \end{array}\right] \rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 8 \end{bmatrix}$$

$x_1 = 4 \quad x_2 = 8$

$$\begin{bmatrix} 3 & -4 & 8 \\ 2 & -5 & -2 \\ \hline 1 & 0 & -6 \end{bmatrix} \qquad \left[\begin{array}{c|cc} 0 & -8 & 1 \\ 5 & 2 & -1 \\ -3 & 3 & 7 \end{array}\right]$$

$$\begin{bmatrix} 0+-20+-24 & -24+-8+24 & \\ & & \end{bmatrix} = \begin{bmatrix} -44 & -8 & 63 \\ -19 & -32 & -7 \\ 18 & -26 & -41 \end{bmatrix}$$

Matrix Multiplication

$$A \times B = AB$$

$m \times n \qquad n \times p \qquad m \times p$

Is matrix multiplication commutative?

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}$$

3×2 $\qquad$ $2 \times 2 \rightarrow 3 \times 2$

row x column

$$\begin{bmatrix} -1+0 & -2+21 \\ 4+0 & 8-35 \\ 0+0 & 0+14 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 19 \\ 4 & -27 \\ 0 & 14 \end{bmatrix}$$