

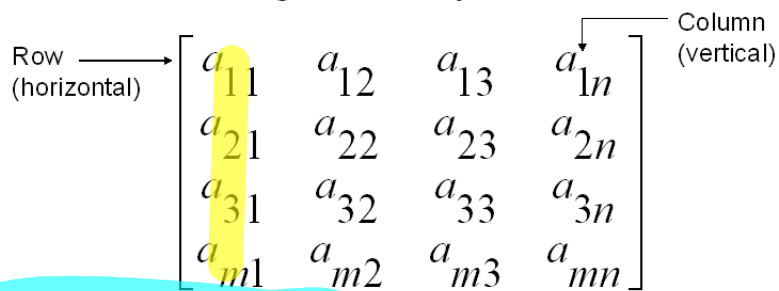
Matrices and Gauss  
Jordan Solving

$$\begin{array}{r} -x + y - z = -14 \\ 2x - y + z = 21 \\ 3x - 2y + z = 19 \end{array}$$



# Matrices!

Matrix-a rectangular array of real numbers



Matrices are in form  $m \times n$  ( $m$  by  $n$ ), where  $m$  is the number of rows and  $n$  is the number of columns.

Entries in the  $i$ th row and  $j$ th column are denoted by  $a_{ij}$  (so  $a_{32}$  is in the 3<sup>rd</sup> row and 2<sup>nd</sup> column)

\* order  
row x columns

## Example 1



Determine the order of each matrix:

a)  $\begin{bmatrix} 3 & 5 \\ 14 & -9 \end{bmatrix}$   
 $2 \times 2$

c)  $\begin{bmatrix} 18 & 7 & 1 \\ -64 & 2 & 0 \end{bmatrix}$   
 $2 \times 3$

b)  $[16 \ 7 \ 1 \ 17]$   
 $1 \times 4$

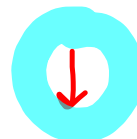
Solve:

$$x - 2y + 3z = 9$$

$$-x + 3y = -4$$

$$2x - 5y + 5z = 17$$

$$\rightarrow \left[ \begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array} \right]$$



Reduced  
Row  
Echelon  
Form

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$x = 1 \quad y = -1 \quad z = 2$$

$$\left[ \begin{array}{ccc|c} 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} \infty \\ \text{sol} \end{array}$$

$$\left[ \begin{array}{ccc|c} 0 & 0 & 0 & 3 \end{array} \right] \begin{array}{l} \text{No} \\ \text{sol.} \end{array}$$

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -2 & 2 \\ 2 & 4 \end{bmatrix}$$

$$A - B =$$

$$5A =$$

59)

$$\begin{cases} -x_1 + x_2 = 4 \\ -2x_1 + x_2 = 0 \end{cases}$$

$$\left[ \begin{array}{cc|c} -1 & 1 & 4 \\ -2 & 1 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 8 \end{array} \right]$$
$$x_1 = 4 \quad x_2 = 8$$

$$\begin{bmatrix} 3 & -4 & 8 \\ 2 & -5 & -2 \\ \hline 1 & 0 & -6 \end{bmatrix} \quad \begin{bmatrix} 0 & -8 & 1 \\ 5 & 2 & -1 \\ -3 & 3 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 0 + -20 + 24 & -24 + -8 + 24 \\ -44 & -8 & 63 \\ -19 & -32 & -7 \\ 18 & -26 & -41 \end{bmatrix} = \begin{bmatrix} -44 & -8 & 63 \\ -19 & -32 & -7 \\ 18 & -26 & -41 \end{bmatrix}$$



## Matrix Multiplication

$$\begin{array}{c} \mathbf{A} \quad \times \quad \mathbf{B} \quad = \quad \mathbf{AB} \\ m \times n \quad n \times p \quad m \times p \end{array}$$

Is matrix multiplication commutative?

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ \underline{0} & \underline{2} \end{bmatrix}$$

$$B = \left[ \begin{array}{cc|c} 1 & 2 & \\ 0 & 7 & \end{array} \right]$$

3x2

2x2 → 3x2

row x  
column

$$\begin{bmatrix} -1+0 & -2+21 \\ 4+0 & 8-35 \\ 0+0 & 0+14 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 19 \\ 4 & -27 \\ 0 & 14 \end{bmatrix}$$

